

The Influence of Modification of Gravity on the Dynamics of Radiating Spherical Fluids

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Abstract

We explore the evolutionary behaviors of compact objects in a modified gravitational theory with the help of structure scalars. Particularly, we consider the spherical geometry coupled with heat and radiation emitting shearing viscous matter configurations. We construct structure scalars by splitting the Riemann tensor orthogonally in $f(R, T)$ gravity with and without constant R and T constraints, where R is the Ricci scalar and T is the trace of the energy-momentum tensor. We investigate the influence of modification of gravity on the physical meaning of scalar functions for radiating spherical matter configurations. It is explicitly demonstrated that even in modified gravity, the evolutionary phases of relativistic stellar systems can be analyzed through the set of modified scalar functions.

Keywords: Structure scalars; Relativistic dissipative fluids; Modified gravity

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1 Introduction

Gravitation is probably easily conceived elementary interaction that one can experience in everyday life. Relativistic study is seen to be the foundation of modern physics along with the quantum theory. In the exploration of celestial weak gravitational interaction, relativistic effects must be taken into account. There are lucid examples of relativistic stellar systems, such as white dwarfs, neutron stars, black holes in which these effects may have major outcomes. As a matter of fact, in order to study these systems, it becomes necessary to take observationally viable gravity theories. Further, many interesting results coming from observational ingredients of Supernovae Ia, cosmic microwave background (CMB) radiation, etc. [1] have made a great revolution in the field of cosmology and gravitational physics thus opening a new research platform. These observations and experiments reveal that currently, there is an accelerating expansion in our cosmos. According to the recent observational results obtained from, e.g., the Planck satellite [2, 3, 4], the BICEP2 experiment [5, 6, 7], and the Wilkinson Microwave anisotropy probe (WMAP) [8, 9], the energy fraction of the baryonic matter is only 5%, while that of dark matter and dark energy are 27% and 68%, respectively.

Introducing modified gravitational theories after generalizing the Einstein-Hilbert (EH) action to explore the mystery of cosmic accelerating expansion is a very popular approach among relativistic astrophysicists. Nojiri and Odintsov [10] explained that why extension to Einstein gravity theories are attractive in exploring the evolutionary mechanism of cosmic late acceleration. Extended gravity theories involve $f(R)$, $f(T)$ etc., where T is the torsion scalar in teleparallel gravity (for further reviews on dark energy and modified gravity, see, for instance, [11]). The simplest extension of the Einstein's theory is $f(R)$ theory obtained by replacing the Ricci scalar with its arbitrary function in the EH action. This theory was brought in after few years from the advent of the Einstein's relativity to analyze possible alternatives [12] and was then studied occasionally by several researchers [13] to renormalize general relativity [14] which requires higher curvature dark source terms in the EH action. This theory attracted many relativistic astrophysicists in the possible explanation of cosmic inflation due to the quadratic Ricci scalar corrections [15] in the EH action.

In the similar fashion, many other extended gravity theories has been discussed, like, $f(G)$ in which G is the Gauss-Bonnet invariant. Other curvature amalgams have also been employed such as $f(R, G)$ and $f(R, R_{\alpha\beta}^{\alpha\beta}, R_{\alpha\beta\gamma\delta}^{\alpha\beta\gamma\delta})$.

Nevertheless, less interest has been noticed to more complex gravity theories. It is worthy to mention that modification in scalar curvature is useful in many ways. When one take the case of low curvature, accelerating cosmic expansion can be observed [16], while the high curvature can be used to smoothen the singularities. In this respect, Harko *et al.* [17] put forward the basis of $f(R)$ gravity and gave the notion of $f(R, T)$ theory (where quantity T is induced by quantum effects or exotic imperfect matter distributions) in which he made matter geometry coupling. They solved dynamical equations interpreting some cosmological and astronomical backgrounds by taking various $f(R, T)$ models.

Houndjo [18] performed cosmological reconstruction in $f(R, T)$ gravity and claimed that his models could possibility unify cosmic accelerated and matter dominated eras. Jamil *et al.* [19] discussed the reconstruction of some well-known astrophysical models with $f(R, T)$ corrections and obtained results consistent with low red-shifts Baryonic Acoustic Oscillations observations. Adhav [20] investigated exact solutions of some cosmological models by taking exponential volumetric expansion in this theory. Baffou *et al.* [21] investigated dynamical evolution along with stability of power law and de-Sitter cosmic models against linear perturbation. They concluded that such models can be considered as a competitive dark energy candidate. Sun and Huang [22] addressed some cosmic issues of isotropic and homogeneous universe in $f(R, T)$ gravity and found results consistent with astronomical observation data.

Anisotropic effects are leading paradigms in addressing the evolutionary mechanisms of celestial imploding models. The assumption of considering isotropic nature of pressure distribution in self-gravitating relativistic bodies is often under discussion by many researchers. However, there are several arguments indicating that the relativistic fluid pressure can be slightly varied in different directions (anisotropic) at any particular point. Bowers and Liang [23] did pioneer work in describing possible significance of locally anisotropic pressure distribution in relativistic spherical matter configurations and found that anisotropy may have worthwhile effects on parameters controlling the hydrostatic equilibrium of celestial systems. The anisotropy within stellar systems can be observed through number of interested interconnected mechanisms, e.g., existence of strong electric and magnetic interactions [24], condensations of pions [25], phase transitions [26], the presence of vacuum core [27], even the emergence of gravitational waves from non-static meridional axial stellar structures [28] etc. It can be demonstrated that the mixture of two

fluid configurations can be mathematically treated as an anisotropic framework. Chakraborty *et al.* [29] investigated pressure anisotropy contributions on the collapsing quasi-spherical model and found that such configurations of pressure could obstruct appearance of naked singularity. The dynamical analysis of a collapsing relativistic stellar system has been performed [30, 31] and it has been shown that the invoking of R^α ($1 < \alpha \leq 2$) corrections could lead to a viable and singularity free model [31].

The characterization of gravitational collapse of stellar interiors under numerous scenarios remain an open research window in relativistic astrophysics. Ghosh and Maharaj [32] investigated dynamics of collapsing dust cloud with $f(R)$ corrections and found relatively stable fluid configurations against perturbation mode. Cembranos *et al.* [33] studied collapsing mechanism of relativistic dust particles and found highly contracted configurations of collapsing systems due to the presence of $f(R)$ gravitational interaction. Capozziello *et al.* [34] calculated modified versions of Poisson and Boltzmann dynamical equations for relativistic self-gravitating structures in $f(R)$ gravity and found that some more unstable modes of the evolving systems at N region due to $f(R)$ dark source terms. Sebastiani *et al.* [35] described evolving phases of spherical relativistic systems with $f(R)$ background and found a wide range of different instability regions for the evolving compact systems. Recently, Yousaf and Bhatti [36] explored that some $f(R)$ model configurations would support more compact cylindrical objects with smaller radii as compared to general relativity (GR).

The dynamics of self-gravitating stellar systems can be addressed with the help of system's structural variables, such as local pressure anisotropy, energy density, Weyl scalar, etc. The density irregularities and anisotropy occupy major character in the collapsing mechanism and thus in developing theory of cosmic structure formation. Any relativistic system begins collapsing once it enters into an inhomogeneous state. Thus, in order to study subsequent evolution of collapsing fluid configurations, one requires to explore factors responsible for producing energy density irregularities. In this perspective, Penrose and Hawking [37] explored irregularities in the energy density of spherical relativistic stars by means of Weyl invariant. Herrera *et al.* [38] evaluated inhomogeneity parameters for anisotropic spherical compact objects and found that local anisotropy may yield appearance of naked singularities. Mena *et al.* [39] analyzed role of shearing and regular appearance of the collapsing dust fluid on the subsequent evolution. Herrera *et al.* [40] related Weyl scalar with fluid parameters and discussed gravitational

arrow of time for dissipative spherical star. Herrera *et al.* [41] explored role of cosmological constant in the irregularity factors and shear and evolution equations. Sharif and his collaborators [42] explored some inhomogeneity and dynamical factors for tilted charged, conformally flat and non-tilted relativistic systems with different backgrounds. Recently, Sharif and Yousaf [43] found energy density irregularity parameters in the subsequent evolution of celestial objects in $f(R)$ gravity.

This paper extended the work [41] in order to describe the effects of modification of Einstein gravity in the formulation of structure scalars. We also explore the role of these scalar variables in the evolution equations for dissipative self-gravitating spherical compact stars. The paper is organized as follows. In section 2, we describe spherical dissipative matter configuration with $f(R, T)$ formalism and then make a connection between structural variables with the Weyl invariant. Section 3 is devoted to formulate modified scalar functions and elaborate their role in the dynamics of self-gravitating systems while section 4 discusses the contribution of these structure variables for dust fluid with constant Ricci and trace of stress-energy tensors. The main findings are concluded in the last section.

2 Spherical Dissipative Fluid Description with $f(R, T)$ Formalism

We consider $f(R, T)$ gravity achieved by generalizing the action of general relativity coupled with ordinary matter Lagrangian L_M as [17]

$$S_{f(R,T)} = \int d^4x \sqrt{-g} [f(R, T) + L_M], \quad (1)$$

where g , T are the traces of metric and usual energy-momentum tensors, respectively while R and L_M represent Ricci scalar and matter Lagrangian density. We chose the unit system, $8\pi G = c = 1$. The usual energy-momentum tensor can be found as

$$T_{\alpha\beta} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_M)}{\delta g^{\alpha\beta}}. \quad (2)$$

If we assume that L_M depends merely on metric variables, i.e., $g_{\alpha\beta}$ (not upon its derivatives), then we have the following form of energy momentum tensor

$$T_{\alpha\beta} = g_{\alpha\beta}L_M - 2\frac{\partial(L_M)}{\partial g^{\alpha\beta}}. \quad (3)$$

Upon varying the modified EH action with respect to $g^{\alpha\beta}$, we get

$$\begin{aligned} \delta S_{f(R,T)} = \int \left\{ f_R \delta(g^{\alpha\beta} R_{\alpha\beta}) - \frac{f}{2} g_{\alpha\beta} \delta g^{\alpha\beta} + f_T \frac{\delta T}{\delta g^{\alpha\beta}} \delta g^{\alpha\beta} \right. \\ \left. + \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_M)}{\delta g^{\alpha\beta}} \right\} \sqrt{-g} d^4x, \end{aligned} \quad (4)$$

where subscripts T and R describe $\frac{\partial}{\partial T}$ and $\frac{\partial}{\partial R}$ operators, respectively. Considering variations of Ricci scalar and Christoffel symbols, the above equation can be recast as

$$\begin{aligned} \delta S_{f(R,T)} = \int \left\{ f_R g_{\alpha\beta} \square \delta g^{\alpha\beta} - f_R \nabla_\alpha \nabla_\beta \delta g^{\alpha\beta} + f_T \frac{\delta(g^{\mu\nu} T_{\mu\nu})}{\delta g^{\alpha\beta}} \delta g^{\alpha\beta} - \frac{f}{2} g_{\alpha\beta} \delta g^{\alpha\beta} \right. \\ \left. + f_R R_{\alpha\beta} \delta g^{\alpha\beta} + \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_M)}{\delta g^{\alpha\beta}} \right\} \sqrt{-g} d^4x, \end{aligned} \quad (5)$$

where ∇_α represents covariant derivation while \square indicates $\nabla_\alpha \nabla^\alpha$ operator. Now, we consider $T \equiv g^{\mu\nu} T_{\mu\nu}$ variations with respect to $g^{\alpha\beta}$ as

$$\frac{\delta(g^{\mu\nu} T_{\mu\nu})}{\delta g^{\alpha\beta}} = T_{\alpha\beta} + \Theta_{\alpha\beta}^1, \quad (6)$$

where

$$\Theta_{\alpha\beta}^1 \equiv g^{\mu\nu} \frac{\delta T_{\mu\nu}}{\delta g^{\alpha\beta}}. \quad (7)$$

Keeping in mind partial integration of first and second terms of Eq.(5), one can obtain the following configurations of $f(R, T)$ field equation as

$$R_{\alpha\beta} f_R - (\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \square) f_R - \frac{f}{2} g_{\alpha\beta} = (1 - f_T) T_{\alpha\beta}^{(m)} - f_T \Theta_{\alpha\beta}^1. \quad (8)$$

Now, we can continue our calculation after substituting the value of $\Theta_{\alpha\beta}^1$ and this is possible once we have matter Lagrangian. Variation of Eq.(3) provides

$$\frac{\delta T_{\mu\nu}}{\delta g^{\alpha\beta}} = L_M \frac{\delta g_{\mu\nu}}{\delta g^{\alpha\beta}} + \frac{L_M}{2} g_{\alpha\beta} g_{\mu\nu} - \frac{g_{\mu\nu}}{2} T_{\alpha\beta} - 2 \frac{\partial^2 L_M}{\partial g^{\alpha\beta} \partial g^{\mu\nu}}. \quad (9)$$

Using this relation in Eq.(7), we obtain

$$\Theta_{\alpha\beta}^1 = L_M g_{\alpha\beta} - 2T_{\alpha\beta} - 2 \frac{\partial^2 L_M}{\partial g^{\alpha\beta} \partial g^{\mu\nu}} g^{\mu\nu}. \quad (10)$$

The choice of matter Lagrangian is directly connected with the value of $\Theta_{\alpha\beta}^1$. As the dynamical equations in this theory depends upon contribution from matter contents, therefore one can obtain particular scheme of equations corresponding to every selection of L_M . For example, for electromagnetic field theory one can take $L_M = -F_{\mu\nu} F_{\zeta\eta} g^{\mu\zeta} g^{\nu\eta}$, (where $F_{\mu\nu}$ is the Maxwell tensor) for which $\Theta_{\alpha\beta}^1 = -T_{\alpha\beta}$. Here, we are considering the even more complex problem in which non-static geometry of spherical system is coupled with shearing viscous and locally anisotropic fluid configurations, radiating through heat flux and free streaming approximation. We assume the following mathematical expression of the stress-energy tensor (along with $L_M = -\mu$)

$$T_{\alpha\beta} = P_{\perp} h_{\alpha\beta} + \mu V_{\alpha} V_{\beta} + \Pi \chi_{\alpha} \chi_{\beta} + \varepsilon l_{\alpha} l_{\beta} + q(\chi_{\beta} V_{\alpha} + \chi_{\alpha} V_{\beta}) - 2\eta \sigma_{\alpha\beta}, \quad (11)$$

where μ is the energy density, q is a scalar quantity corresponding to a heat conducting vector, q_{β} . The quantity q_{β} can be expressed by means of radial unit four vector, $\chi_{\beta} = H\delta_{\beta}^1$, as

$$q_{\beta} = q\chi_{\beta}.$$

Further, η is the coefficient of shear viscosity, while ϵ and $\sigma_{\alpha\beta}$ are radiation density and shear tensor, respectively. Moreover, $h_{\alpha\beta} = g_{\alpha\beta} + V_{\alpha} V_{\beta}$ is the projection tensor and Π is the difference of radial, P_r , and tangential pressure, P_{\perp} , given by $\Pi \equiv P_r - P_{\perp}$. Now with the help of Eq.(12), we obtain

$$\Theta_{\alpha\beta}^1 = -2T_{\alpha\beta} - \mu g_{\alpha\beta}. \quad (12)$$

The corresponding $f(R, T)$ field equations are given as follows

$$G_{\alpha\beta} = T_{\alpha\beta}^{\text{eff}}, \quad (13)$$

where

$$T_{\alpha\beta}^{\text{eff}} = \left[(1 + f_T(R, T)) T_{\alpha\beta} + \mu g_{\alpha\beta} f_T(R, T) + \left(\frac{f(R, T)}{R} - f_R(R, T) \right) \frac{R}{2} g_{\alpha\beta} \right. \\ \left. + (\nabla_{\alpha} \nabla_{\beta} - g_{\alpha\beta} \square) f_R(R, T) \right] \frac{1}{f_R(R, T)}$$

is the effective energy-momentum tensor encapsulating gravitational contribution coming from $f(R, T)$ extra degrees of freedom while $G_{\alpha\beta}$ is the Einstein tensor.

We consider non-static geometry of spherical system

$$ds^2 = -A^2(t, r)dt^2 + H^2(t, r)dr^2 + C^2d\theta^2 + C^2\sin^2\theta d\phi^2, \quad (14)$$

where A , H are dimension-less quantities while C has L dimension. The quantities V^β and l^β in Eq.(11) are fluid four-velocity and the null four-vector, respectively. The four-vectors $V^\beta = \frac{1}{A}\delta_0^\beta$, χ^β , $l^\beta = \frac{1}{A}\delta_0^\beta + \frac{1}{H}\delta_1^\beta$, and $q^\beta = q(t, r)\chi^\beta$ under co-moving coordinates obey

$$\begin{aligned} V^\alpha V_\alpha &= -1, & \chi^\alpha \chi_\alpha &= 1, & \chi^\alpha V_\alpha &= 0, \\ V^\alpha q_\alpha &= 0, & l^\alpha V_\alpha &= -1, & l^\alpha l_\alpha &= 0. \end{aligned}$$

The kinematical scalars representing expansion and shearing motion of spherical symmetric metric are given, respectively, as follows

$$\Theta = \frac{1}{A} \left(\frac{2\dot{C}}{C} + \frac{\dot{H}}{H} \right), \quad \sigma = \frac{1}{A} \left(\frac{\dot{H}}{H} - \frac{\dot{C}}{C} \right),$$

where the over dot represents $\frac{\partial}{\partial t}$ operation.

The $f(R, T)$ field equations for spherically relativistic interior system (with signatures $(-1, 1, 1, 1)$) are

$$G_{00} = \frac{A^2}{f_R} \left[\mu + \varepsilon - \frac{R}{2} \left(\frac{f}{R} - f_R \right) + \frac{\psi_{00}}{A^2} \right], \quad (15)$$

$$G_{01} = \frac{AH}{f_R} \left[-(1 + f_T)(q + \varepsilon) + \frac{\psi_{01}}{AH} \right], \quad (16)$$

$$G_{11} = \frac{H^2}{f_R} \left[\mu f_T + (1 + f_T)(P_r + \varepsilon - \frac{4}{3}\eta\sigma) + \frac{R}{2} \left(\frac{f}{R} - f_R \right) + \frac{\psi_{11}}{H^2} \right], \quad (17)$$

$$G_{22} = \frac{C^2}{f_R} \left[(1 + f_T)(P_\perp + \frac{2}{3}\eta\sigma) + \mu f_T + \frac{R}{2} \left(\frac{f}{R} - f_R \right) + \frac{\psi_{22}}{C^2} \right], \quad (18)$$

where

$$\psi_{00} = 2\partial_{tt}f_R + \left(\frac{\dot{H}}{H} - 2\frac{\dot{A}}{A} + 2\frac{\dot{C}}{C} \right) \partial_t f_R + \left(A^2 \frac{H'}{H} - 2AA' - 2A^2 \frac{C'}{C} \right) \frac{\partial_r f_R}{H^2},$$

$$\begin{aligned}
\psi_{01} &= \partial_t \partial_r f_R - \frac{A'}{A} \partial_t f_R - \frac{\dot{H}}{H} \partial_r f_R, \\
\psi_{11} &= \partial_{rr} f_R - \frac{H^2}{A^2} \partial_{tt} f_R + \left(H^2 \frac{\dot{A}}{A} - 2H^2 \frac{\dot{C}}{C} - 2H \dot{H} \right) \frac{\partial_t f_R}{A^2} \\
&\quad + \left(\frac{A'}{A} + 2 \frac{C'}{C} - 2 \frac{H'}{H} \right) \partial_r f_R, \\
\psi_{22} &= -C^2 \frac{\partial_{tt} f_R}{A^2} + \frac{C^2}{A^2} \left(\frac{\dot{A}}{A} - 3 \frac{\dot{C}}{C} - \frac{\dot{H}}{H} \right) \partial_t f_R + \frac{C^2}{H^2} \left(\frac{C'}{C} + \frac{A'}{A} - \frac{H'}{H} \right) \partial_r f_R.
\end{aligned}$$

Here, the prime indicates $\frac{\partial}{\partial r}$ operation.

The four-velocity of the relativistic collapsing fluid, U , can be obtained by taking variations of areal radius of spherical systems with its proper time as follows

$$U = D_T C = \frac{\dot{C}}{A}. \quad (19)$$

The Misner-Sharp mass function $m(t, r)$ is given by [44]

$$m(t, r) = \frac{C}{2} \left(1 + \frac{\dot{C}^2}{A^2} - \frac{C'^2}{H^2} \right). \quad (20)$$

By making use of using Eqs. (15)–(17), (19) and (20), the variation of spherical mass function with respect to time and radius can be given, respectively, as follows

$$\begin{aligned}
D_T m &= \frac{-1}{2f_R} \left[U \left\{ (1 + f_T)(\bar{P}_r - \frac{4}{3}\eta\sigma) + \mu f_T + \frac{R}{2} \left(\frac{f}{R} - f_R \right) + \frac{\psi_{11}}{H^2} \right\} \right. \\
&\quad \left. + E \left\{ (1 + f_T)\bar{q} - \frac{\psi_{01}}{AH} \right\} C^2 \right], \quad (21)
\end{aligned}$$

$$D_C m = \frac{C^2}{2f_R} \left[\bar{\mu} - \frac{R}{2} \left(\frac{f}{R} - f_R \right) + \frac{\psi_{00}}{A^2} + \frac{U}{E} \left\{ (1 + f_T)\bar{q} - \frac{\psi_{01}}{AH} \right\} \right]. \quad (22)$$

where $D_C = \frac{1}{C'} \frac{\partial}{\partial r}$, $\bar{P}_r = P_r + \varepsilon$, $\bar{\mu} = \mu + \varepsilon$ and $\bar{q} = q + \varepsilon$. The Integration of Eq.(22) gives

$$m = \frac{1}{2} \int_0^C \frac{C^2}{f_R} \left[\bar{\mu} - \frac{R}{2} \left(\frac{f}{R} - f_R \right) + \frac{\psi_{00}}{A^2} + \frac{U}{E} \left\{ \frac{(1 + f_T)}{f_R} \bar{q} - \frac{\psi_{01}}{AH} \right\} \right] dC, \quad (23)$$

where $E \equiv \frac{C'}{H}$. This can be expressed with the help of Eq.(19) as

$$E \equiv \frac{C'}{H} = \left[1 + U^2 - \frac{2m(t, r)}{C} \right]^{1/2}. \quad (24)$$

This specific combinations of dissipation structural variables, energy density and $f(R, T)$ corrections through mass function can be achieved from Eq.(23) and is given as

$$\frac{3m}{C^3} = \frac{3\kappa}{2C^3} \int_0^r \left[\bar{\mu} - \frac{R}{2} \left(\frac{f}{R} - f_R \right) + \frac{\psi_{00}}{A^2} + \frac{U}{E} \left\{ \frac{(1 + f_T)}{f_R} \bar{q} - \frac{\psi_{01}}{AH} \right\} C^2 C' \right] dr. \quad (25)$$

The well-known couple of components of the Weyl tensor are defined as

$$E_{\alpha\beta} = C_{\alpha\phi\beta\varphi} V^\phi V^\varphi, \quad H_{\alpha\beta} = \tilde{C}_{\alpha\gamma\beta\delta} V^\gamma V^\delta = \frac{1}{2} \epsilon_{\alpha\gamma\eta\delta} C^{\eta\delta}_{\beta\rho} V^\gamma V^\rho,$$

where $\epsilon_{\alpha\beta\gamma\delta} \equiv \sqrt{-g} \eta_{\alpha\beta\gamma\delta}$ with $\eta_{\alpha\beta\gamma\delta}$ is a Levi-Civita symbol, while $E_{\alpha\beta}$ and $H_{\alpha\beta}$ represent electric and magnetic Weyl tensor components, respectively. The electric component, $E_{\alpha\beta}$ in view of unit four velocity and four vectors can be given by

$$E_{\alpha\beta} = \mathcal{E} \left[\chi_\alpha \chi_\beta - \frac{1}{3} (g_{\alpha\beta} + V_\alpha V_\beta) \right],$$

where

$$\begin{aligned} \mathcal{E} &= \left[\frac{\ddot{C}}{C} + \left(\frac{\dot{H}}{H} - \frac{\dot{C}}{C} \right) \left(\frac{\dot{C}}{C} + \frac{\dot{A}}{A} \right) - \frac{\ddot{H}}{H} \right] \frac{1}{2A^2} - \frac{1}{2C^2} \\ &- \left[\frac{C'''}{C} - \left(\frac{C'}{C} + \frac{H'}{H} \right) \left(\frac{A'}{A} - \frac{C'}{C} \right) - \frac{A''}{A} \right] \frac{1}{2H^2} \end{aligned} \quad (26)$$

is the Weyl scalar. This scalar after using Eqs.(20) and (25) can be written as

$$\begin{aligned} \mathcal{E} &= \frac{1}{2f_R} \left[\bar{\mu} - (1 + f_T)(\bar{\Pi} - 2\eta\sigma) - \frac{R}{2} \left(\frac{f}{R} - f_R \right) + \frac{\psi_{00}}{A^2} - \frac{\psi_{11}}{H^2} + \frac{\psi_{22}}{C^2} \right] \\ &- \frac{3}{2C^3} \int_0^r \frac{C^2}{f_R} \left[\bar{\mu} - \frac{R}{2} \left(\frac{f}{R} - f_R \right) + \frac{\psi_{00}}{A^2} + \frac{U}{E} \left\{ \frac{(1 + f_T)}{f_R} \bar{q} - \frac{\psi_{01}}{AH} \right\} \right. \\ &\quad \left. \times C^2 C' \right] dr, \end{aligned} \quad (27)$$

where $\bar{\Pi} = \bar{P}_r - P_\perp$. Here we have assumed a regular matter configuration at the center, i.e., $m(t, 0) = 0 = C(t, 0)$. The above expression provides a link between Weyl scalar, $f(R, T)$ higher curvature quantities and structural variables of matter distribution (pressure anisotropy, radiation and energy density, heat radiating vector, shearing viscosity).

3 Modified Scalar Variables and $f(R, T)$ Gravity

In this section, we firstly take a viable configuration of $f(R, T)$ model and then construct scalar functions after orthogonally splitting Reimman tensor. In order to present $f(R, T)$ gravity as an acceptable theory, one should consider viable as well as well-consistent $f(R, T)$ models. Thus, we investigate the dynamical properties of dissipating anisotropic spherical fluid distribution by taking following configuration of $f(R, T)$ model [45]

$$f(R, T) = f_1(R) + f_2(T). \quad (28)$$

This choice yields a minimal matter curvature coupling, thereby presenting $f(R, T)$ gravity as corrections of $f(R)$ gravity. Starobinsky [15] suggested that quadratic Ricci scalar corrections, i.e., $f(R) = R + \alpha R^2$ in the field equations could be helpful to cause exponential early universe expansion. Several relativisticists [46] adopted this formulation not only for an inflationary constitute but also as a substitute for dark matter (DM) for $\alpha = \frac{1}{6M^2}$ [47]. For DM model, M is figured out as $2.7 \times 10^{-12} \text{GeV}$ with $\alpha \leq 2.3 \times 10^{22} \text{GeV}^{-2}$ [48]. This is the minimum value of M which follows from Cavendish-type laboratory tests of the Newton law of gravity. It is interesting to mention that extension to this model could provide a platform different from that of $R + \alpha R^2$ to understand various cosmic puzzles. Here, we take $f_1(R) = R + \alpha R^n - \beta R^{2-n}$ [49] along with $f_2(T) = \lambda T$, with α , β and λ as positive real numbers. The particular selection of $f_1(R)$ could be constructive to discuss inflation from the αR^n along with a stable minimum of the scalar potential of an auxiliary field. This also assists to obtain a potential having a non-zero residual vacuum energy, thereby providing it as a DE in the late-time cosmic evolution.

In order to present acceptable $f(R, T)$ theory of gravity, one should consider viable as well as well-consistent $f(R, T)$ models. A viable model not

only helps to shed light over current cosmic acceleration but also obeys the requirements imposed by terrestrial and solar system experiments with relativistic background. Further, they should satisfy minimal constraints for theoretical viability. Any modified gravity model needs to possess exact cosmological dynamics and avoids the instabilities such as ghosts (Dolgov-Kawasaki instability, Ostrogradski's instability and tachyons). The following conditions should be satisfied for a viable $f_1(R)$ models [50]:

- The positive value of $f_{1R}(R)$ with $R > \tilde{R}$, here \tilde{R} is the today value of the Ricci invariant. This condition is needed to avoid appearance of a ghost state. Ghost often appears, while dealing with modified gravity theories that informs DE as a source behind current cosmic acceleration. This may be induced due to a mysterious force which is repulsive in nature between massive or supermassive stellar objects at large distances. The constraint of keeping effective gravitational constant, $G_{eff} = G/f_{1R}$, to be positive is also significant to retain the attractive feature of gravity.
- The positive value of $f_{1RR}(R)$ with $R > \tilde{R}$. This condition is introduced to avoid the emergence of tachyons. A tachyon is any hypothetical particle traveling faster than speed of light. The moving mass of these particles would be imaginary and one can assume the imaginary rest-mass so that moving mass must be real.

If $f_1(R)$ models do not satisfy these conditions, then it would be regarded as unviable. Haghani *et al.* [51] and Odintsov and Saez-Gomez [52] suggested that Dolgov-Kawasaki instability in $f(R, T)$ gravity requires similar sort of limitations as in $f(R)$ gravity and in addition to this, we require $1 + f_T > 0$ for $G_{eff} > 0$. So, for the viable $f(R, T)$ models, one needs to satisfy the constraints

$$f_R > 0, 1 + f_T > 0, f_{RR} > 0, R \geq \tilde{R}.$$

It is worthy to stress that in our chosen $f(R, T)$ form, the term $1 + \lambda$ is explicitly taken to be positive. One thing that needs to be emphasized here is that the divergence of energy-momentum tensor is non-zero in $f(R, T)$ gravity (unlike GR) and is found as

$$\nabla^\alpha T_{\alpha\beta} = \frac{f_T}{1 - f_T} \left[(\Theta_{\alpha\beta} + T_{\alpha\beta}) \nabla^\alpha \ln f_T - \frac{1}{2} g_{\alpha\beta} \nabla^\alpha T + \nabla^\alpha \Theta_{\alpha\beta} \right]. \quad (29)$$

This makes breaking of all equivalence principles in $f(R, T)$ gravity. The weak equivalence principle states “*All test particles in a given gravitational field, will undergo the same acceleration, independent of their properties, including their rest mass*”. But in $f(R, T)$ gravity, the equation of motion depends on the thermodynamic properties of the particles (energy density and pressure etc). The strong equivalence principle, “*The gravitational motion of a small test body depends only on its initial position and velocity, and not on its constitution*” is again obviously broken because of the non-geodesic motion of the particles. The dynamics of $f(R)$ gravity can be recovered, on setting $f(T) = 0$.

Following Bel [53] and Herrera *et al.* [40, 41], we introduce couple of following tensors $Y_{\alpha\beta}$ and $X_{\alpha\beta}$ as

$$Y_{\alpha\beta} = R_{\alpha\gamma\beta\delta} V^\gamma V^\delta, \quad X_{\alpha\beta} = {}^* R_{\alpha\gamma\beta\delta}^* V^\gamma V^\delta = \frac{1}{2} \eta^{\epsilon\rho}{}_{\alpha\gamma} R_{\epsilon\rho\beta\delta}^* V^\gamma V^\delta,$$

where $R_{\alpha\beta\gamma\delta}^* = \frac{1}{2} \eta_{\epsilon\rho\gamma\delta} R^{\epsilon\rho}{}_{\alpha\beta}$. In order to develop formalism for structure scalars in $f(R, T)$ gravity, we orthogonally decomposed Riemann curvature tensor. and found that

$$X_{\alpha\beta} = X_{\alpha\beta}^{(m)} + X_{\alpha\beta}^{(D)} = \frac{1}{3f_R} \left[\bar{\mu} - \frac{R}{2} \left(\frac{f}{R} - f_R \right) + \frac{\psi_{00}}{A^2} \right] h_{\alpha\beta} - \frac{1}{2f_R} \left[(1 + \lambda)(\bar{\Pi} - 2\eta\sigma) + \frac{\psi_{11}}{H^2} - \frac{\psi_{22}}{C^2} \right] \left(\chi_\alpha \chi_\beta - \frac{1}{3} h_{\alpha\beta} \right) - E_{\alpha\beta} \quad (30)$$

$$Y_{\alpha\beta} = Y_{\alpha\beta}^{(m)} + Y_{\alpha\beta}^{(D)} = \frac{1}{6f_R} \left[\bar{\mu} + 3\mu\lambda + (1 + \lambda)(3P_r - 2\bar{\Pi}) + \frac{\psi_{00}}{A^2} + \frac{\psi_{11}}{H^2} + \frac{2\psi_{22}}{C^2} + R \left(\frac{f}{R} - f_R \right) \right] h_{\alpha\beta} + \frac{1}{2f_R} \left[(1 + \lambda)(\bar{\Pi} - 2\eta\sigma) + \frac{\psi_{11}}{H^2} - \frac{\psi_{22}}{C^2} \right] \times \left(\chi_\alpha \chi_\beta - \frac{1}{3} h_{\alpha\beta} \right) - E_{\alpha\beta}. \quad (31)$$

These tensors can be written as a combination of their trace and trace-less components as follows

$$X_{\alpha\beta} = \frac{1}{3} Tr X h_{\alpha\beta} + X_{<\alpha\beta>}, \quad (32)$$

$$Y_{\alpha\beta} = \frac{1}{3} Tr Y h_{\alpha\beta} + Y_{<\alpha\beta>}, \quad (33)$$

where

$$X_{<\alpha\beta>} = h_\alpha^\rho h_\beta^\gamma \left(X_{\rho\gamma} - \frac{1}{3} \text{Tr} X h_{\rho\gamma} \right), \quad (34)$$

$$Y_{<\alpha\beta>} = h_\alpha^\rho h_\beta^\gamma \left(Y_{\rho\gamma} - \frac{1}{3} \text{Tr} Y h_{\rho\gamma} \right). \quad (35)$$

From Eqs.(28)-(31), we found

$$\begin{aligned} \text{Tr} X \equiv X_T &= \frac{1}{1 + n\alpha R^{n-1} - \beta(2-n)R^{1-n}} \left\{ \bar{\mu} - \frac{\alpha(1-n)}{2} R^n + \frac{\beta(3-n)}{2} \right. \\ &\times \left. R^{2-n} - \frac{\lambda}{2} T + \frac{\hat{\psi}_{00}}{A^2} \right\}, \end{aligned} \quad (36)$$

$$\begin{aligned} \text{Tr} Y \equiv Y_T &= \frac{1}{2(1 + n\alpha R^{n-1} - \beta(2-n)R^{1-n})} \left\{ \bar{\mu} + 3\mu\lambda + 3(1+\lambda)\bar{P}_r \right. \\ &- 2(1+\lambda)\bar{\Pi} + \frac{\hat{\psi}_{00}}{A^2} + \frac{\hat{\psi}_{11}}{H^2} + \frac{2\hat{\psi}_{22}}{C^2} + 2\alpha(1-n)R^n + 2\beta(3-n)R^{2-n} - 2\lambda T \Big\}, \end{aligned} \quad (37)$$

where hat indicates that the corresponding dark source terms are evaluated after using $f(R, T)$ model. We can also write $X_{<\alpha\beta>}$ and $Y_{<\alpha\beta>}$ in an alternatively form

$$X_{<\alpha\beta>} = X_{TF} \left(\chi_\alpha \chi_\beta - \frac{1}{3} h_{\alpha\beta} \right), \quad (38)$$

$$Y_{<\alpha\beta>} = Y_{TF} \left(\chi_\alpha \chi_\beta - \frac{1}{3} h_{\alpha\beta} \right), \quad (39)$$

where the quantities X_{TF} and Y_{TF} are

$$\begin{aligned} X_{TF} &= -\mathcal{E} - \frac{1}{2(1 + n\alpha R^{n-1} - \beta(2-n)R^{1-n})} \left\{ (1+\lambda)(\bar{\Pi} - 2\sigma\eta) \right. \\ &\quad \left. + \frac{\hat{\psi}_{11}}{H^2} - \frac{\hat{\psi}_{22}}{C^2} \right\}, \end{aligned} \quad (40)$$

$$\begin{aligned} Y_{TF} &= \mathcal{E} - \frac{1}{2(1 + n\alpha R^{n-1} - \beta(2-n)R^{1-n})} \left\{ (\bar{\Pi} - 2\eta\sigma)(1+\lambda) \right. \\ &\quad \left. + \frac{\hat{\psi}_{11}}{H^2} - \frac{\hat{\psi}_{22}}{C^2} \right\}. \end{aligned} \quad (41)$$

The scalar function Y_{TF} can be written in terms of matter variables after using Eqs.(27), (34) and (37) as

$$\begin{aligned}
Y_{TF} = & \frac{1}{2(1 + n\alpha R^{n-1} - \beta(2-n)R^{1-n})} \left(\bar{\mu} - 2(1+\lambda)(\bar{\Pi} - 4\eta\sigma) + \frac{\alpha}{2} \right. \\
& \times (1-n)R^n - \frac{\beta}{2}(3-n)R^{2-n} + \frac{\lambda}{2}T + \frac{\hat{\psi}_{00}}{A^2} - \frac{2\hat{\psi}_{11}}{H^2} + \frac{2\hat{\psi}_{22}}{C^2} \left. \right) - \frac{3}{2C^3} \\
& \times \int_0^r \frac{C^2}{1 + n\alpha R^{n-1} - \beta(2-n)R^{1-n}} \left[\bar{\mu} - \frac{\alpha}{2}(1-n)R^n + \frac{\beta}{2}(3-n)R^{2-n} \right. \\
& \left. - \frac{\lambda}{2}T + \frac{\hat{\psi}_{00}}{A^2} + \frac{U}{E} \left\{ \frac{(1+\lambda)}{1 + n\alpha R^{n-1} - \beta(2-n)R^{1-n}} \bar{q} - \frac{\hat{\psi}_{01}}{AH} \right\} C^2 C' \right] dr.
\end{aligned} \tag{42}$$

Now we express fluid variables by defining some effective variables

$$\begin{aligned}
\mu_{eff} &\equiv \bar{\mu} + \frac{\hat{\psi}_{00}}{A^2}, \quad P_r^{eff} \equiv \bar{P}_r + \frac{\hat{\psi}_{11}}{H^2} - \frac{4}{3}\eta\sigma, \\
P_{\perp}^{eff} &\equiv P_{\perp} + \frac{\hat{\psi}_{22}}{C^2} + \frac{2}{3}\eta\sigma, \\
\Pi^{eff} &\equiv P_r^{eff} - P_{\perp}^{eff} = \Pi - 2\eta\sigma - \frac{\psi_{22}}{C^2} + \frac{\psi_{11}}{H^2}.
\end{aligned}$$

These terms are just like the usual matter structure variables with the difference that they have modified gravity as well as viscosity terms in some specific combination. Using above effective variables, Eqs.(36), (37), (40) and (41) reduce to

$$\begin{aligned}
X_{TF} = & \frac{3\kappa}{2C^3} \int_0^r \left[\frac{1}{\{1 + n\alpha R^{n-1} - \beta(2-n)R^{1-n}\}} \left\{ \mu_{eff} - \frac{\alpha}{2}(1-n)R^n + \frac{\beta}{2} \right. \right. \\
& \times (3-n)R^{(2-n)} - \frac{\lambda}{2}T + \left(\hat{q} - \frac{\psi_q}{AB} \right) \frac{U}{E} \left. \right\} C^2 C' \right] dr \\
& - \frac{1}{2\{1 + n\alpha R^{n-1} - \beta(2-n)R^{1-n}\}} \left[\mu_{eff} - \frac{\alpha}{2}(1-n)R^n + \frac{\beta}{2}(3-n) \right. \\
& \left. \times R^{(2-n)} - \frac{\lambda}{2}T \right],
\end{aligned} \tag{43}$$

$$Y_{TF} = \frac{1}{2(1 + n\alpha R^{n-1} - \beta(2-n)R^{1-n})} \left[\mu_{eff} - \frac{\alpha}{2}(1-n)R^n + \frac{\beta}{2} \right]$$

$$\begin{aligned}
& \times (3-n)R^{(2-n)} - \frac{\lambda}{2}T - 2(1+\lambda)\Pi^{eff} + 2\lambda \left(\frac{\hat{\psi}_{11}}{H^2} - \frac{\hat{\psi}_{22}}{C^2} \right) \Bigg] \\
& - \frac{3}{2C^3} \int_0^r \left[\frac{1}{\{1+n\alpha R^{n-1} - \beta(2-n)R^{1-n}\}} \left\{ \mu_{eff} - \frac{\alpha}{2}(1-n)R^n \right. \right. \\
& \left. \left. + \frac{\beta}{2}(3-n)R^{(2-n)} - \frac{\lambda}{2}T + \left(\hat{q} - \frac{\psi_q}{AH} \right) \frac{U}{E} \right\} C^2 C' \right] dr, \tag{44}
\end{aligned}$$

$$\begin{aligned}
Y_T &= \frac{1}{2(1+n\alpha R^{n-1} - \beta(2-n)R^{1-n})} [(1+3\lambda)\mu_{eff} - 3\varepsilon\lambda + 3(1+\lambda) \\
& \times P_r^{eff} - 2(1+\lambda)\Pi^{eff} - \lambda \left(\frac{\psi_{11}}{H^2} + 3\frac{\psi_{00}}{A^2} \right) + 2(2+\lambda)\frac{\psi_{22}}{C^2} - 2\alpha \\
& \times (1-n) - 2\beta(3-n)R^{(2-n)} - 2\lambda T], \tag{45}
\end{aligned}$$

$$\begin{aligned}
X_T &= \frac{1}{(1+n\alpha R^{n-1} - \beta(2-n)R^{1-n})} \left[\mu_{eff} - \frac{\alpha}{2}(1-n)R^n + \frac{\beta}{2}(3-n) \right. \\
& \left. \times R^{2-n} - \frac{\lambda}{2}T \right]. \tag{46}
\end{aligned}$$

On setting $\lambda = 0$, the $f(R)$ structure [43] scalars can be retrieved from the above expressions. These structure functions have a direct correspondence with the dynamical evolution of relativistic compact systems even in $f(R, T)$ gravity theory. It is evident from Eq.(46) that X_T has foremost importance in the definition of stellar energy density along with the effects of dark source terms coming from $f(R, T)$ gravity. Following Herrera *et al.* [41], the evolution equation connecting effects of tidal forces with fluid parameters variables is

$$\begin{aligned}
& \left[X_{TF} + \frac{1}{2\{1+n\alpha R^{n-1} - \beta(2-n)R^{1-n}\}} \left\{ \bar{\mu}_{eff} - \frac{\alpha(1-n)}{2}R^n + \frac{\beta}{2}(3-n) \right. \right. \\
& \left. \left. \times R^{2-n} - \frac{\lambda}{2}T \right\} \right]' = -X_{TF} \frac{3C'}{C} + \frac{(\sigma - \Theta)}{2[1+n\alpha R^{n-1} - \beta(2-n)R^{1-n}]} \left[\frac{\psi_{01}}{H} \right. \\
& \left. - \frac{(1+\lambda)}{[1+n\alpha R^{n-1} - \beta(2-n)R^{1-n}]} \bar{q}H \right]. \tag{47}
\end{aligned}$$

It is evident from the above equation that in the absence of dark source and radiating variables, one can obtain the following result after considering regularity constraints as

$$\mu'_{eff} = 0 \Leftrightarrow X_{TF} = 0.$$

This means that X_{TF} controls the inhomogeneity of the collapsing star. In the non-radiating isotropic matter distribution, we can get from Eq.(47), that μ'_{eff} only exists if and only if \mathcal{E} exists. This suggests that tidal forces try to move the self-gravitating compact objects into inhomogeneous window as the time proceeds. This led Penrose to describe a gravitational time arrow with the help of the Weyl tensor in GR. To check the role of rest of structure variables, we consider well-known mathematical tool put forward through the so-called Raychaudhuri equation (also calculated individually by Landau) [54]. In view of one of the modified structure scalars, it follows that

$$-Y_T = V^\alpha \Theta_{;\alpha} + \frac{2}{3} \sigma^{\alpha\beta} \sigma_{\alpha\beta} + \frac{\Theta^2}{3} - a^\alpha_{;\alpha}. \quad (48)$$

This relation shows that Y_T has an utmost relevance in the description of the expansion rate of self-gravitating relativistic fluids. The equation describing the shear evolution can be recast in terms of Y_{TF} as follows

$$Y_{TF} = a^2 + \chi^\alpha a_{;\alpha} - \frac{aC'}{HC} - \frac{2}{3} \Theta \sigma - V^\alpha \sigma_{;\alpha} - \frac{1}{3} \sigma^2, \quad (49)$$

thus describing that $f(R, T)$ correction terms have its importance in the shearing motion of the evolving relativistic spherical self-gravitating system.

4 Evolution Equations with Constant R and T

Here, we discuss the contribution of modified structure scalars for the dust spherical cloud with Ricci scalar and $T \equiv T^\beta_\beta$ background. In this context, the quantity of matter within the spherical model of radius r is

$$m = \frac{1}{2\{1 + n\alpha\tilde{R}^{n-1} - \beta(2-n)\tilde{R}^{1-n}\}} \int_0^r (\mu) C^2 C' dr - \frac{\alpha(1-n)\tilde{R}^n - \beta(3-n)\tilde{R}^{2-n} + \lambda\tilde{T}}{2\{1 + n\alpha\tilde{R}^{n-1} - \beta(2-n)\tilde{R}^{1-n}\}} \int_0^r C^2 C' dr, \quad (50)$$

where tilde indicates that the corresponding terms are evaluated under constant backgrounds. For the dust interior cloud, the couple of equations describing the tidal forces and peculiar form of mass function can be given as

follows

$$\mathcal{E} = \frac{1}{2C^3\{1 + n\alpha\tilde{R}^{n-1} - \beta(2-n)\tilde{R}^{1-n}\}} \int_0^r \mu' C^3 dr - \frac{\alpha(1-n)\tilde{R}^n - \beta(3-n)\tilde{R}^{2-n} + \lambda\tilde{T}}{4\{1 + n\alpha\tilde{R}^{n-1} - \beta(2-n)\tilde{R}^{1-n}\}}, \quad (51)$$

$$\frac{3m}{C^3} = \frac{1}{2\{1 + n\alpha\tilde{R}^{n-1} - \beta(2-n)\tilde{R}^{1-n}\}} \left[\mu - \frac{1}{C^3} \int_0^r \mu' C^3 dr \right] + \frac{\alpha(1-n)\tilde{R}^n - \beta(3-n)\tilde{R}^{2-n} + \lambda\tilde{T}}{2\{1 + n\alpha\tilde{R}^{n-1} - \beta(2-n)\tilde{R}^{1-n}\}}. \quad (52)$$

These equations are equivalent to Eqs.(25) and (28). The $f(R, T)$ scalar functions in the realm of constant Ricci and T turn out to be

$$\tilde{X}_T = \frac{1}{\{1 + n\alpha\tilde{R}^{n-1} - \beta(2-n)\tilde{R}^{1-n}\}} \left[\mu - \frac{\alpha}{2}(1-n)\tilde{R}^n = \frac{\beta}{2}(3-n) \times \tilde{R}^{2-n} - \frac{\lambda}{2}\tilde{T} \right], \quad \tilde{Y}_{TF} = -\tilde{X}_{TF} = \mathcal{E}, \quad (53)$$

$$\tilde{Y}_T = \frac{1}{2\{1 + n\alpha\tilde{R}^{n-1} - \beta(2-n)\tilde{R}^{1-n}\}} \left[\mu + 3\mu\lambda - 2\alpha(1-n)\tilde{R}^n + 2\beta \times (3-n)\tilde{R}^{2-n} - 2\lambda\tilde{T} \right], \quad (54)$$

where tilde indicates that the quantities are computed under constant Ricci scalar condition. It is clear from Eq.(53) that X_T describes matter energy density in the mysterious dark universe while the evolution equation that describes the behavior of the regular energy density over the relativistic dust cloud, can be expressed by means of X_{TF} as follows

$$\left[\frac{\mu}{2\{1 + n\alpha\tilde{R}^{n-1} - \beta(2-n)\tilde{R}^{1-n}\}} - \frac{\alpha(1-n)\tilde{R}^n - \beta(3-n)\tilde{R}^{2-n} + \lambda\tilde{T}}{4\{1 + n\alpha\tilde{R}^{n-1} - \beta(2-n)\tilde{R}^{1-n}\}} + \tilde{X}_{TF} \right]' = -\frac{3}{C} \tilde{X}_{TF} C'. \quad (55)$$

It follows from the above equation that the constant $f(R, T)$ corrections controls the irregularity in the energy density of the matter distribution. The scenario $\mu' = 0$ is directly related with the vanishing of \tilde{X}_{TF} . This reinforces the importance of structure scalar, X_{TF} in the modeling of self-gravitating

matter configurations. The shear and expansion evolution equations can be written in terms of rest of structure scalars as follows

$$V^\alpha \Theta_{;\alpha} + \frac{2}{3}\sigma^2 + \frac{\Theta^2}{3} - a_{;\alpha}^\alpha \frac{1}{\{1 + n\alpha\tilde{R}^{n-1} - \beta(2-n)\tilde{R}^{1-n}\}} \left[\mu - \frac{\alpha}{2}(1-n) \right. \\ \left. \times \tilde{R}^n = \frac{\beta}{2}(3-n)\tilde{R}^{2-n} - \frac{\lambda}{2}\tilde{T} \right] = -\tilde{Y}_T, \quad (56)$$

$$V^\alpha \sigma_{;\alpha} + \frac{\sigma^2}{3} + \frac{2}{3}\sigma\Theta = -\mathcal{E} = -\tilde{Y}_{TF}. \quad (57)$$

5 Conclusions

In the present paper, we have discussed the dynamical properties of compact objects by taking into account the well-known $f(R, T)$ high degrees of freedom. First, we have studied spherical self-gravitating system coupled with relativistic viscous matter distribution that is radiating with free streaming out and diffusion approximations. After evaluating the basic formulae, we have related the system structural variables to the Weyl scalar. We have then investigated factors that affect the contribution of tidal forces in the evolution of collapsing spherical matter distribution in the realm of $f(R, T)$ gravity. In order to bring out the effects of modified gravity corrections, we have considered a particular class of $f(R, T)$ models, i.e., the form is given by $f(R, T) = f_1(R) + f_2(T)$. This choice does not imply the direct non-minimal curvature matter coupling. Nevertheless, it can be regarded as a correction to $f(R)$ gravity. We have used the linear form of f_2 and acquired distinct results on the basis of a non-trivial coupling in comparison with $f(R)$ gravity.

We have explored the role of $f(R, T)$ dark source terms in the expressions of structure scalars. These scalars have been obtained from the orthogonal decomposition of the Riemann curvature tensor. We have found that as in general relativity, these scalar variables controls the evolutionary mechanisms of radiating fluid spheres in cosmos and they are four in number.

Our main results are summarized as follows.

(i) It is found from Eq.(36) that one of the structure scalar (which is the trace part of Eq.(30)) describes the energy density (along dark source terms) of dissipative anisotropic spherical distribution. This also shows that $f(R, T)$ correction affects the contribution of X_T due to its non-attractive nature.

(ii) It is well-known from the working of [55] that structure scalar, Y_T , has a direct link with Tolman mass “density” for dynamical systems to be in

the phases of equilibrium/quasi-equilibrium. We found that the role of this quantity is controlled by anisotropic pressure along with dark energy/matter terms. More appropriately, we can say that Tolman mass density is conspicuously related with pressure anisotropy, radiating and non-radiating energy density along $f(R, T)$ correction. It is seen from Eq.(37) that even in radiating spheres, Y_T can have a direct link with non-dissipative energy density due to the presence of $3\mu\lambda$ (this term comes due to $f(R, T)$ gravity). Thus $f(R, T)$ gravity enhances the contribution of energy density in the description of Tolman mass.

(iii) The expansion scalar has utmost relevance with vacuum core emergence within the stellar interior (see [57]). Further, it is well-known that expansion-free constraint requires pressure anisotropy. It is seen from Eqs. (48) and (56) that the evolution of expansion scalar is fully controlled by Y_T . Thus, Y_T may be helpful to understand the emergence of vacuum cavity within the celestial object. This sparks that Y_T should have a direct relation with pressure anisotropy along with $f(R, T)$ corrections and this is obvious from Eqs. (45).

(iv) The structure scalar, Y_{TF} depicts the influence of both local pressure anisotropy, shear viscosity along with tidal forces in the mysterious dark universe as seen from Eq. (41). Furthermore, this scalar variable fully control the shear evolution expression as mentioned by Eq.(49). Thus, in order to understand the role the shear motion on the dynamical phases of the radiating celestial object, one needs to study the behavior of Y_{TF} .

(v) Any celestial system must experience inhomogeneous state in order to move into the collapsing phase. We found that the quantity that controls irregularities in the energy density of stellar interior is X_{TF} . It is well-known from the working of [40] that X_{TF} behaves as an inhomogeneity factor for dust perfect and anisotropic fluid. However, Eq.(47) describes that dissipative parameters as well as $f(R, T)$ corrections tend to produce hindrances in the role of X_{TF} . However, if one considers that expansion scalar is proportional to shear scalar, then it is only $f(R, T)$ degrees of freedom that tends to produce hindrances in the appearance of inhomogeneities in the relativistic self-gravitating systems. Thus, X_{TF} along with dark source terms control density irregularities, and thus should be the basic ingredient in the definition of a gravitational arrow of time.

(vi) For constant curvature background with dust cloud matter distribution, it is seen that it is the tidal forces (that are expressed with the help of X_{TF}) which are responsible for producing irregularities in the initial homo-

geneous stellar system.

(vii) All of our results are reduced to those in Ref. [41] by taking $f(R, T) = R$.

Consequently, it has been shown that $f(R, T)$ gravity tends to lessen down density homogeneity, which in turn induce the stability to the relativistic collapsing compact systems. The $f(R, T)$ gravity gives a new corrections to the EH Lagrangian through the coupling of matter and geometry. In this theory, the cosmic acceleration depends not only on a geometrical contribution to the total cosmic energy density but also on the cosmic matter. If one takes $f(R, T) = R + \lambda T$, then one can obtain dynamics identical with that of GR. Thus, the simple case $f(R, T) = R + \lambda T$ is fully equivalent with standard GR, after rescaling of λ .

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References

- [1] D. Pietrobon, A. Balbi, and D. Marinucci, Phys. Rev. D **74**, 043524 (2006); T. Giannantonio, *et al.*, Phys. Rev. D **74**, 063520 (2006); A. G. Riess, *et al.*, Astrophys. J. **659**, 98 (2007).
- [2] Planck collaboration, Ade, P. A. R. *et al.*: Astron. Astrophys. **571**, A1 (2014).
- [3] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1502.01589 [astro-ph.CO].
- [4] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1502.02114 [astro-ph.CO].

- [5] P. A. R. Ade *et al.* [BICEP2 Collaboration], Phys. Rev. Lett. **112**, 241101 (2014) [arXiv:1403.3985 [astro-ph.CO]].
- [6] P. A. R. Ade *et al.* [BICEP2 and Planck Collaborations], Phys. Rev. Lett. **114**, 101301 (2015) [arXiv:1502.00612 [astro-ph.CO]].
- [7] P. A. R. Ade *et al.* [BICEP2 and Keck Array Collaborations], arXiv:1510.09217 [astro-ph.CO].
- [8] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **192**, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]].
- [9] G. Hinshaw *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **208**, 19 (2013) [arXiv:1212.5226 [astro-ph.CO]].
- [10] Nojiri, S. and Odintsov, S.D.: eConf C **0602061** (2006) 06 [Int. J. Geom. Meth. Mod. Phys. **4**, (2007) 115] [hep-th/0601213].
- [11] S. Nojiri and S. D. Odintsov, Phys. Rept. **505**, 59 (2011) [arXiv:1011.0544 [gr-qc]]; A. Joyce, B. Jain, J. Khoury and M. Trodden, Phys. Rept. **568**, 1 (2015) [arXiv:1407.0059 [astro-ph.CO]]; S. Capozziello and V. Faraoni, *Beyond Einstein Gravity* (Springer, Dordrecht, 2010); S. Capozziello and M. De Laurentis, Phys. Rept. **509**, 167 (2011) [arXiv:1108.6266 [gr-qc]]; K. Bamba, S. Capozziello, S. Nojiri and S. D. Odintsov, Astrophys. Space Sci. **342**, 155 (2012) [arXiv:1205.3421 [gr-qc]]; K. Koyama, arXiv:1504.04623 [astro-ph.CO]; A. de la Cruz-Dombriz and D. Sáez-Gómez, Entropy **14**, 1717 (2012) [arXiv:1207.2663 [gr-qc]]; K. Bamba, S. Nojiri and S. D. Odintsov, arXiv:1302.4831 [gr-qc]; K. Bamba and S. D. Odintsov, arXiv:1402.7114 [hep-th]; Symmetry **7**, 220 (2015) [arXiv:1503.00442 [hep-th]].
- [12] A. S. Eddington, : *The Mathematical Theory of Relativity*. (Cambridge University Press, 1923); H. J. Schmidt, Int. J. Geom. Meth. Phys. **4**, 209 (2007).
- [13] C. Lanczos, Zeit. Phys. **73**, 147 (1932); E. Schrödinger, *Space-Time Structure*, (Cambridge University Press, 1950); H. Buchdhal, Quart. J. Math. **19**, 150 (1948); Proc. Nat. Acad. Sci. **34**, 66 (1948); Acta Mathematica **85**, 63 (1951); J. London Math. Soc. **26**, 139 (1951); Mon. Not. R. Astron. Soc. **150**, 1 (1970); J. Phys. A **11**, 871 (1978); Int. J. Theor. Phys. **17**, 149 (1978).

- [14] R. Utiyama and B. DeWitt, J. Math. Phys. **3**, 608 (1962); A. Strominger, Phys. Rev. D **30**, 2257 (1984).
- [15] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980).
- [16] K. Bolejko *et al.*, *Structures in the Universe by Exact Methods- Formation, Evolution, Interactions* (Cambridge University Press, 2009).
- [17] T. Harko, F. S. N. Lobo, S. Nojiri and S. D. Odintsov, Phys. Rev. D **84**, 024020 (2011).
- [18] M. J. S. Houndjo, Int. J. Mod. Phys. D **21**, 1250003 (2012).
- [19] M. Jamil, D. Momeni, M. Raza and R. Myrzakulov, Eur. Phys. J. C **72**, 1999 (2012).
- [20] K. S. Adhav, Astrophys. Space Sci. **339**, 365 (2012).
- [21] E. H. Baffou, A. V. Kpadonou, M. E. Rodrigues, M. J. S. Houndjo and J. Tossa, Astrophys. Space Sci. **356**, 173 (2014).
- [22] G. Sun and Y. Huang, arXiv:1510.01061.
- [23] R. Bowers and E. Liang, Astrophys. J. **188**, 657 (1974).
- [24] S. S. Yazadjiev, Phys. Rev. D **85**, 044030 (2012).
- [25] R. Sawyer, Phys. Rev. Lett. **29**, 382 (1972).
- [26] A. Sokolov, J. Exp. Theor. Phys. **79**, 1137 (1980).
- [27] M. Sharif and Z. Yousaf, Mon. Not. R. Astron. Soc. **440**, 3479 (2014); *ibid.* Eur. Phys. J. C **75**, 58 (2015).
- [28] L. Herrera, A. Di Prisco, J., Ibàñez and J. Ospino, Phys. Rev. D **89**, 084034 (2014); M. Sharif and Z. Yousaf, Eur. Phys. J. C **75**, 194 (2015).
- [29] S. Chakraborty, S. Chakraborty and U. Debnath, Int. J. Mod. Phys. D **14**, 1707 (2005).
- [30] E. V. Arbuzova and A. D. Dolgov, Phys. Lett. B **700** (2011) 289 doi:10.1016/j.physletb.2011.05.030 [arXiv:1012.1963 [astro-ph.CO]].

- [31] K. Bamba, S. Nojiri and S. D. Odintsov, Phys. Lett. B **698**, 451 (2011).
- [32] S. G. Ghosh and S. D. Maharaj, Phys. Rev. D **85**, 124064 (2012).
- [33] J. A. R. Cembranos, A. de la Cruz-Dombriz and B. M. Núñez, J. Cosmol. Astropart. Phys. **04**, 021 (2012).
- [34] S. Capozziello, M. D. Laurentis, I. D. Martino, M. Formisano and S. D. Odintsov, Phys. Rev. D **85**, 044022 (2012).
- [35] L. Sebastiani, D. Momeni, R. Myrzakulov and S. D. Odintsov, Phys. Rev. D **88**, 104022 (2013).
- [36] Z. Yousaf and M. Z. Bhatti, Mon. Not. Roy. Astron. Soc. doi: 10.1093/mnras/stw423.
- [37] R. Penrose and S. W. Hawking, *General Relativity, An Einstein Centenary Survey*, Cambridge University Press, Cambridge (1979)
- [38] L. Herrera, A. Di Prisco, J. L. Hernández-Pastora and N. O. Santos, Phys. Lett. A **237**, 113 (1998).
- [39] F. C. Mena, R. Tavakol and P. S. Joshi, Phys. Rev. D **62**, 044001 (2000).
- [40] L. Herrera, A. Di Prisco, J. Martin, J. Ospino, N. O. Santos and O. Troconis, Phys. Rev. D **69**, 084026 (2004).
- [41] L. Herrera, A. Di Prisco and J. Ibáñez, Phys. Rev. D **84**, 107501 (2011).
- [42] M. Sharif and M. Z. Bhatti, Int. J. Mod. Phys. D **23**, 1450085 (2014); ibid. **24**, 1550014 (2015); Mod. Phys. Lett. A **29**, 1450165 (2014); ibid. **29**, 1450094 (2014); ibid. **29**, 1450129 (2014); Astrophys. Space Sci. **349**, 995 (2014); ibid. **352**, 883 (2014); M. Sharif and Z. Yousaf, Astrophys. Space Sci. **352**, 321 (2015); ibid. **354**, 481 (2014); ibid. **357**, 49 (2015); Eur. Phys. J. C **75**, 194 (2015); Can. J. Phys. **93**, 905 (2015).
- [43] M. Sharif and Z. Yousaf, Eur. Phys. J. C **75**, 58 (2015).
- [44] C. W. Misner and D. Sharp, Phys. Rev. **136**, B571 (1964).
- [45] M. J. S. Houndjo and O. F. Piattella, Int. J. Mod. Phys. D **21**, 1250024 (2012).

- [46] S. Capozziello, S. Nojiri and S. D. Odintsov, Phys. Lett. B **632**, 597 (2006); A. Borowiec, W. Godłowski and M. Szydłowski, Int. J. Geom. Meth. Mod. Phys. **4**, 183 (2007); S. Capozziello, V. F. Cardone and A. Troisi, Mon. Not. R. Astron. Soc. **375**, 1423 (2007); A. Kelleher, arXiv:1309.3523.
- [47] J. A. R. Cembranos, Phys. Rev. Lett. **102**, 141301 (2009); J. Phys. Conf. Ser. **315**, 012004 (2011).
- [48] J. A. R. Cembranos, A. D. L. Cruz-Dombriz and B. M. Núñez, J. Cosmol. Astropart. Phys. **04**, 021 (2012).
- [49] M. Artymowski and L. Lalak, J. Cosmol. Astropart. Phys. **09**, 036 (2014).
- [50] S. Capozziello, Int. J. Mod. Phys. D **11**, 483 (2002); S. Nojiri and S. D. Odintsov, Phys. Rep. **505**, 59 (2011); K. Bamba et al., Astrophys. Space Sci. **342**, 155 (2012).
- [51] Z. Haghani, T. Harko, F. S. N. Lobo, H. R. Sepangi and S. Shahidi, Phys. Rev. D **88**, 044023 (2013).
- [52] S. D. Odintsov and D. Saez-Gomez, Phys. Lett. B **725**, 437 (2013).
- [53] L. Bel, Ann. Inst. H Poincaré **17**, 37 (1961).
- [54] F. D. Albareti, J. A. R. Cembranos, A. de la Cruz-Dombriz and A. Dobado, J. Cosmol. Astropart. Phys. **03**, 012 (2014).
- [55] L. Herrera, J. Ospino, A. Di Prisco, E. Fuenmayor and O. Troconis, Phys. Rev. D **79**, 064025 (2009).
- [56] L. Herrera, J. Ospino, A. Di Prisco, E. Fuenmayor and O. Troconis, Phys. Rev. D **79**, 064025 (2009).
- [57] L. Herrera, N. O. Santos, and A. Wang, Phys. Rev. D **78**, 084026 (2008); L. Herrera, G. Le Denmat, and N. O. Santos, Phys. Rev. D **79**, 087505 (2009); M. Sharif and Z. Yousaf, Phys. Rev. D **88**, 024020 (2013).